HW 9.3: Line integrals, the fundamental theorem & applications

Determine if $\mathbf{F}$ is conservative. If it is, find its potential function $f$.

1. $\mathbf{F}(x,y) = (2x-3y, -3x+4y-8)$
2. $\mathbf{F}(x,y) = (e^s\sin y, e^s\cos y)$

The follow fields are conservative. Use that fact to evaluate the line integral of $\mathbf{F}$ along $C$.

3. $\mathbf{F}(x,y) = (2xy^2, 2x^2y)$ and $C$ is $r=(t+\sin(0.5\pi t), t+\cos(0.5\pi t))$ for $0<t<1$
4. $\mathbf{F}(x,y,z) = (yz,xz,xy+2z)$ and $C$ is the straight line segment from $(1,0,-2)$ to $(4,6,3)$
5. For the vector field $\mathbf{F}=(2y^{3/2}, 3xy^{1/2})$
   a) Find the work done by $\mathbf{F}$ to move an object from $(1,1)$ to $(2,4)$
   b) what property of $\mathbf{F}$ makes it possible to answer this question?
6. For the wire lying on the curve $r=(t+\sin t, t^2-\cos t)$ for $2<t<3$ with density $p(x,y)=xe^y$, write the integrals for the mass $m$, x-moment $M_x$, y-moment $M_y$ [set up, do not solve]
7. Find the work done by the gravitational field of the sun on the earth when the earth moves from its furthest distance to its closest distance. The furthest distance is $1.52\times10^{11}$ m, the closest distance is $1.47\times10^{11}$ m, the sun mass is $1.99\times10^{30}$ kg, the earth mass is $5.97\times10^{24}$ kg, and $G=6.67\times10^{-11}$ N m$^2$/kg$^2$.

8. The figure below shows a curve $C$ on a contour map of a function $f$. Find $\int_C \nabla f \cdot d\mathbf{r}$

9. The figure below shows the vector field $\mathbf{F}=(2xy, x^2)$ and three curves from $(1,2)$ to $(3,2)$
   a) explain why $\int_C \mathbf{F} \cdot d\mathbf{r}$ has the same value for all three curves.
   b) what is the value?

10. Suppose an experiment determines that the amount of work required for a force field $\mathbf{F}$ to move a particle from the point $(5,3)$ to the point $(1,2)$ along a curve $C_1$ is $1.2$ J and the work done by $\mathbf{F}$ in moving the particle along another curve $C_2$ between the same two points is $1.4$ J. What can you say about $\mathbf{F}$? Why?