HW 6.3 – Chain Rule & Implicit Differentiation

Find \( \frac{dz}{dt} \) a) as a general formula b) at the given t-value

1. \( z = x^3y^2 \ldots x = \cos t \) & \( y = \sin t \) \ldots \( t = \pi/3 \)
2. \( z = \arctan(xy) \ldots x = t^2 \) & \( y = \ln t \) \ldots \( t = 1 \)
3. \( z = xe^{y/x} \ldots x = 1-t \) & \( y = 3-2t \) \ldots \( t = 2 \)

Find \( \frac{dy}{dx} \)

4. \( y \arctan x + x^4 \sqrt[4]{2} - \sin y = 0 \)
5. \( \frac{xy}{1 + x^2 + y^2} = e^{x^2} \)

Find the requested partial derivatives

6. \( x^2y^4 + 5x^2z^3 = 4xy + y^2z^2 \ldots \) find \( \frac{\partial z}{\partial y} \) and \( \frac{\partial x}{\partial y} \)
7. \( e^{xz} - yz = 0 \ldots \) find \( \frac{\partial z}{\partial y} \) and \( \frac{\partial y}{\partial x} \)

Applications

8. A manufacturing company's yearly production is worth \( P = 1.47L^{0.65}K^{0.35} \), where \( L \) is the amount of labor (in thousands of hours) and \( K \) is the amount of capital (in million $) and \( P \) is measured in millions of $. When \( L = 30 \) and \( K = 8 \), the labor is decreasing at 2 thousand hours/year and \( K \) is increasing at 0.5 million $/year. Find \( \frac{dP}{dt} \) (in million $/year).